INPUT CONDUCTANCE AND CURRENT DISTRIBUTION OF LONG HORIZONTAL WIRE ANTENNAS ABOVE THE EARTH'S SURFACE

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Abstract

Closed form expressions for computing the input conductance and current distribution of long horizontal antennas near the earth are derived. The theory is an extension of the recently developed theory of modal propagation on an infinite horizontal wire. In that work, a previously unreported propagating mode is shown to be an important part of the current excited on a horizontal wire by a voltage source. It is shown that long antennas have certain advantages when used in geologic remote sensing applications.

Summary

A number of solutions to the horizontal antenna problem have been reported. The approximate techniques reported to date require that the antenna be a significant fraction of a wavelength above a highly conducting earth. The exact solution has been presented but involves a tedious numerical process which is suitable for antennas less than one wavelength in size. No presently available solutions are useful for antennas longer than one wavelength.

The work to be described is valid for antennas which are longer than a half-wavelength (in many cases it is valid for much shorter antennas). The approximate solution given is valid for antennas closer to the earth than permitted by any other than the exact theory. It should be noted that an approximation is now available which will permit the extension of this theory to antennas which are electrically very close to the earth [1].

The problem solved involves an antenna of radius a (meters) and finite length 2ℓ excited at $x = 0$ with a delta function generator of voltage $V \exp(-i\omega t)$ where ω is the radian frequency (Fig.1). It is located at a distance d above an earth with refractive index $n = [\varepsilon_g (1 + i\delta)/\varepsilon_o]^{1/2}$ with δ (loss tangent) given as $\sigma_g/\omega\varepsilon_g$ where ε_g, σ_g, ε_o are respectively the permittivity of the earth, the conductivity of the earth and the permittivity of free space.

The integral equation for the current on the wire antenna can be formulated in a straightforward manner and is given in equation 1.

$$\int_{-\ell}^{\ell} I(x') \, M(x-x') \, dx' = V \delta (x-\ell) \qquad (1)$$

The kernel of the integral equation, $M(x-x')$, involves Sommerfeld integrals through which the earth's effect is introduced and is discussed further by Olsen and Chang [2]. An approximation to the kernel is made which is equivalent to replacing the earth by a surface impedance. The results are valid for $d \gg a$, $n \gg 1$ and $2k_o dn > 1$ where k_o is the wave-number of free space.

The current distribution on long antennas is developed in a manner which directly follows the work of Shen et al. [3] for long antennas in free space. First, a solution to equation 1 is found for the infinitely long antenna ($\ell \to \infty$). This solution has been obtained analytically in closed form through the use of the convolution theorem. The current is composed of four parts. Two can be identified as discrete propagating modes which are bound to the wire. The first of these two modes is the well known transmission line mode. For a perfectly conducting earth it reduces to the TEM wave known to exist on transmission lines. It is a slow wave since the real part of its wavenumber is larger than k_o. The second mode is previously unreported in the literature. It is a fast wave and is attenuated at a significantly smaller rate than is the transmission line mode. Thus at large distances from the source the total current on the wire is dominated by current in this mode. Both currents have been shown to contribute significantly to the total current near the source. The other two parts of the current contribute to the radiation fields and will be discussed. The total current on the infinite antenna is designated as $I_\infty(x)$.

Next, an integral equation is formulated for the case of the current $I_\infty(x)$ being incident upon the end of a semi-infinite horizontal wire above the earth. Using Wiener-Hopf techniques, the semi-infinite integral equation associated with this problem can be solved analytically. A closed form expression for the current reflected from the end of the wire is found. For points on the wire which are not very close to the end, the reflected current has the simple expression

$$I_R(x) \simeq R \, I_\infty(x) \qquad (2)$$

The details of the closed form expression for R will be presented. The use of equation (2) for $I_R(x)$ limits the theory to long antennas. However, the restriction is much less severe than the one for free space antenna theory. In all cases the theory is valid for half-wavelength antennas and in most cases the lower limit is much smaller. Once the reflection properties of the end of the antenna are known the solution for the current distribution on a finite antenna can be obtained. The procedure utilizes the end reflection properties and is the same as the standard techniques used in solving terminated transmission line problems. The current on the antenna shown in Fig. 1 can be expressed as

$$I\ (x) = I_\infty\ (x) + C[I_\infty\ (x-\ell) + I_\infty(x + \ell)] \quad -\ell<x<\ell$$

where

$$C = \frac{R\ I_\infty(\ell)}{1 - R\ I_\infty(2\ell)}\ . \qquad (3)$$

From this current, the input conductance is obtained as

$$G_{IN} = \text{Real}\ [I(x = 0)/V]$$

Curves have been generated for the input conductance of long horizontal antennas as a function of height above the ground (see Fig. 2). As expected, these values oscillate about the input conductance of an antenna in free space of the same length [4]. Additional data will be presented which will illustrate the advantages of using long antennas for geologic remote sensing purposes.

References

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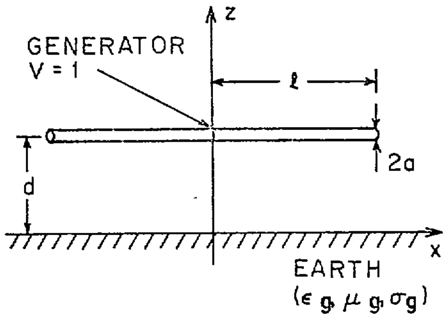

Fig. 1

Geometry of the Finite Antenna

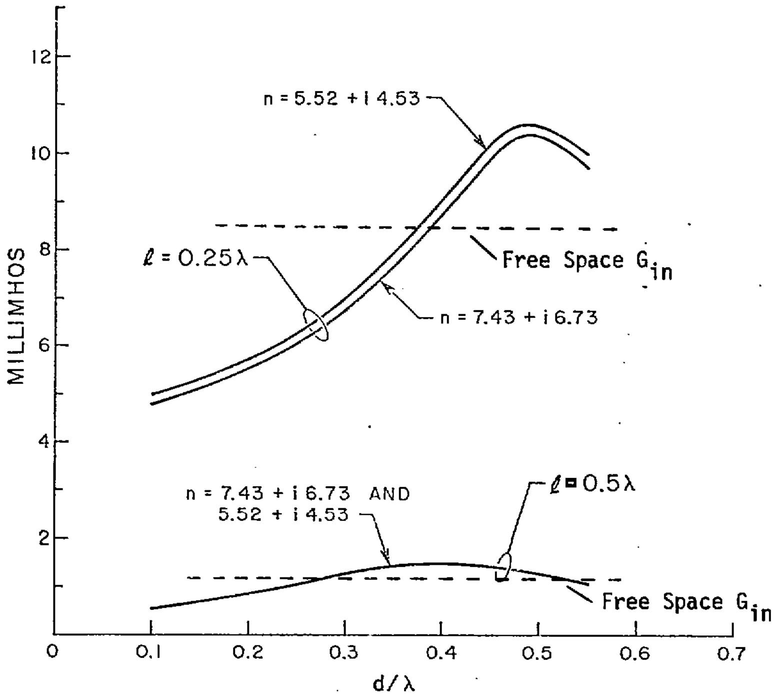

Fig. 2

Input Conductances of Resonant and Anti-resonant Antennas